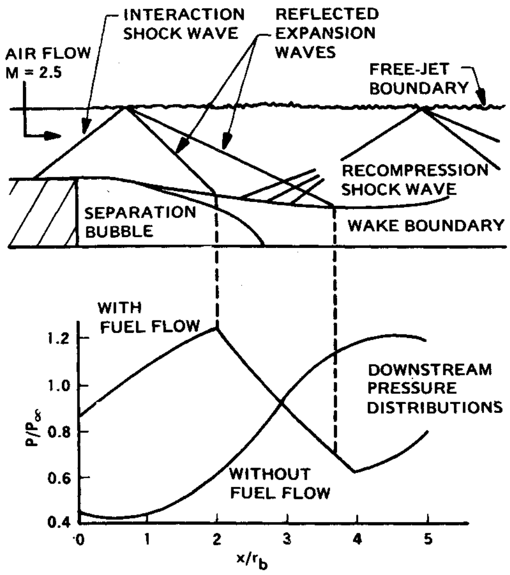

Fig. 5 Interaction shock wave reflection and relation to downstream pressure distributions.

There is no simple solution to this problem other than increasing the nozzle gap width so the leading reflection intersects the wake centerline further than five model radii downstream from the model base. This criterion is easily stated but not easily applied, for prediction of the position and the angle of the leading expansion wave is difficult. Both these parameters are influenced by the fuel flow rate, the nature of the fuel, and the intensity of combustion. Both the position and the angle of the shock wave influence the path of the leading reflected expansion wave.

Test results were satisfactory when an annular nozzle gap of 2.5 in. was used with a 2.5 in. radius model and a Mach 2.0 freestream. However, the gap width cannot be scaled down with model radius because the injection shock wave is curved. As the gap width is reduced proportionally to model radius, the distance between the base and the point of intersection of the shock wave with the freejet boundary decreases more than proportionally to the radius, and the angle of the shock wave at the intersection point increases. Both these effects cause the leading reflected expansion wave to intersect the wake closer to the model base

Conclusions

The external burning propulsion concept can be adequately tested in a small test facility having limited air flow capabilities. The use of a half model appears adequate, but the configuration design requires care to ensure that flow disturbances resulting from fuel injection or a pressure mismatch do not adversely affect the experimental results.

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Viscous Effects in Massively Ablating Planetary Entry Body Flowfields

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Introduction

THE large surface ablation rates caused by severe radiation-dominated re-entry heating can significantly alter the boundary layer and hence the heat-transfer and aerodynamic characteristics of planetary entry vehicles, requiring a sound theoretical understanding of strong ablation-blowing effects. An important but heretofore-untreated fluid mechanical aspect of this problem is the nonsimilar flow development *downstream* of the stagnation point, particularly with regard to the thickening of the mixing layer between the ablation and freestream gases, its influence on the pressure distribution, and its possible impingement on the surface with high local heating which terminates the strong blowing regime. As a first step toward providing a complete theory, the present Note describes an approximate engineering treatment of this problem. Complete details are given in Ref. 1.

Outline of Analysis

The problem considered is flow around a massively ablating two-dimensional or axisymmetric hypersonic blunt-nosed body at zero angle of attack (Fig. 1). To simplify the analysis, the radiation terms are neglected and the flow is assumed to be a nonreacting homogeneous perfect gas mixture with a Prandtl number of unity and a constant density-viscosity ($\rho\mu$) product. Furthermore, the Reynolds number is presumed sufficiently large and the blowing rates moderate enough to permit the use of boundary-layer theory and the neglect of transverse curvature effects by using the Mangler approximation $r \simeq r_w(x)$. Although the laminar flow case is treated here, this three-layer model approach can also be applied to turbulent flows. Another important approximation is neglect of the explicit pressure gradient effects on the solution to the shear layer equations in an appropriately transformed coordinate system. This is known to yield reasonably good engineering solutions for pressure distribution and heat transfer around highly cooled hypersonic blunted bodies including mass transfer.[2] Finally, the local inviscid flowfield is approximated by a Newtonian relationship between the inviscid pressure and the local effective body shape defined by

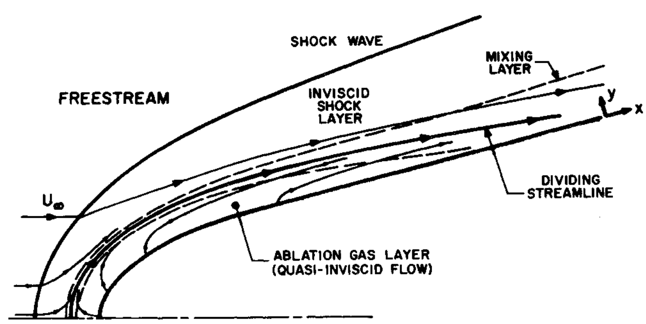

Fig. 1 Massive blowing flow configuration.

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the dividing streamline between the blown and freestream gas. Further discussion of the aforementioned approximations can be found in Ref. 1.

The analysis is then carried out in the body-oriented coordinate system x, y with u and v denoting the respective velocity components. Guided by the results of a matched asymptotic analysis of the massive blowing regime and physical arguments, an approximate "triple-deck" flowfield model of quite general applicability was devised as follows (see Fig. 1). Near the body is a layer of quasi-inviscid rotational blown gas flow in which the total head losses along streamlines are small and hence may be conveniently estimated (see below). Overlaying this inner region is the blown-off laminar (or turbulent) mixing layer flow which is governed by boundary-layer-type equations. It is matched to the inner layer along some location $y_m(x)$ (itself determined as part of the solution) by requiring continuous velocity components, shear stress, profile curvature, temperature, and heat transfer. Finally, the outermost inviscid flow layer of freestream gas is coupled to the inner solution through the total displacement effect of the blown gas layer. Assuming quasi-isentropic motion within the inner blown gas layer, the velocity field within the region is approximately governed by

$$u^2(x,\psi) \approx u_{\mathrm{ref}}^2(x,\psi) + \frac{2\gamma p(x^*)}{(\gamma-1)\rho(x^*)}\left[1-\frac{p(x)}{p(x^*)}\right]^{(\gamma-1)/\gamma} \tag{1}$$

along any streamline ψ originating normal to the body surface with blowing velocity v_w at $x = x^*$, where the (small) viscous total head loss is estimated from some known reference solution u_{ref} appropriate to a blown-off boundary-layer flow (as discussed elsewhere[1] the locally-similar Emmons and Leigh[3] limiting blow-off solution was used in the laminar case). The inclusion of this term permits consideration of possible shear layer impingement downstream. A similar expression can be written for the corresponding quasi-adiabatic temperature field. In the overlaying blown-off shear layer an integral-method analysis was carried out, allowing for a nonzero initial thickness and both heat-transfer and mass flow entrainment from below to permit matching with the foregoing inner blown gas layer solution. Following application of the Levy-Lees compressibility transformation from y to Y and x to ξ and thereafter neglecting the explicit pressure gradient effects, one obtains the following momentum integral:

$$u_e\frac{d}{d\xi}\int_{Y_m}^{Y_e}\left(\frac{u}{u_e}-\frac{u^2}{u_e^{\,2}}\right)dY \simeq \frac{(\partial u/\partial Y)_m}{u_e} + \left(V_m - u_m\frac{dY_m}{d\xi}\right)\left(1-\frac{u_m}{u_e}\right) \tag{2}$$

where Δ is the shear layer thickness and the last term here is nonvanishing because $Y_m(x)$ is in general not a streamline.

The foregoing considerations plus the desire for a reasonably tractible analytical formulation lead to the choice of a quadratic velocity profile in terms of the relative nondimensional coordinate $\zeta = (Y-Y_m)/(Y_e - Y_m)$, although more complicated profiles can also be used. Requiring that $u - u_e$ and $\partial u/\partial Y$ each vanish at $\zeta = 1$ while $u = u_m$ and $\partial u/\partial Y = (\partial u/\partial Y)_m$ equal their corresponding inner solution values from Eq. (1) at $\zeta = 0$ yields a trio of equations (one differential and two algebraic) for the three unknowns Δ, Y_m, and u_m/u_e. Starting with initial values from stagnation point massive blowing theory, Eq. (2) can be numerically integrated downstream to obtain the shear layer thickness growth and velocity at the effective inner edge of the shear layer for any desired surface mass transfer rate distribution. Shear layer impingement, if it occurs, is identified by the condition $u_m \simeq 0$. The dividing streamline y_{DSL}, which is taken to define the effective shape seen by the outer inviscid flow, is determined by an injectant mass conservation equation; the induced pressure field is then given by

$$p_e/p_s \simeq \cos^2\theta_{\mathrm{eff}} = \cos^2[\theta - \tan^{-1}(dy_{DSL}/dx)] \tag{3}$$

where p_s is stagnation pressure.

Illustrative Example

Some preliminary numerical results are now presented for a blunted cone case to show the effects of both the injection level and its distribution. The effects of blowing on the dividing streamline location relative to the body surface (i.e., the blown gas layer thickness) are shown in Fig. 2 in terms of the nondimensional distance $\bar{y} = yRe_s^{1/2}/R_N$, where Re_s is a characteristic stagnation point blowing Reynolds number. The influence of the blowing distribution is shown for a fixed stagnation region injection level λ_B. It is seen that $\bar{y}_{DSL}$ is small and grows slowly near the front of the nose but increases rapidly downstream. When blowing is confined to the nose, the blown gas layer thickness decreases slowly along the afterbody whereas continued injection along the cone causes further growth (tending toward a straight shape for uniform blowing). These curves show how sensitive the blown gas layer thickness and its axial variation can be to the surface mass transfer distribution. Indeed, one can clearly observe the possibility of downstream shear layer impingement ($y_{DSL} \to 0$) when the local blowing rate decays sufficiently rapidly along the afterbody (in the turbulent case with a much faster shear layer spreading, impingement would occur sooner). On the other hand, the present laminar cases show no shear layer impingement when strong blowing continues along the afterbody because the parabolic lateral spreading rate of the laminar free shear layer is much slower than the roughly linear growth in the blowing layer thickness (this conclusion, however, may change in the turbulent case). The corresponding induced pressures are shown in Fig. 3. It is seen that when blowing is confined to the nose region and is not too large ($\lambda_B \leq 1$), the downstream aftereffect is negligible as regards pressure. However, continuation of blowing along the afterbody produces appreciable pressure increases along the body, owing to the larger displacement effect shown in Fig. 2. These preliminary results suggest that the pressure field changes from large blowing rates

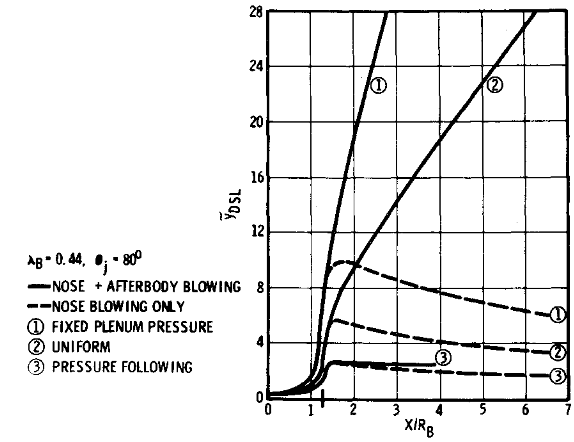

Fig. 2 Dividing streamline location.

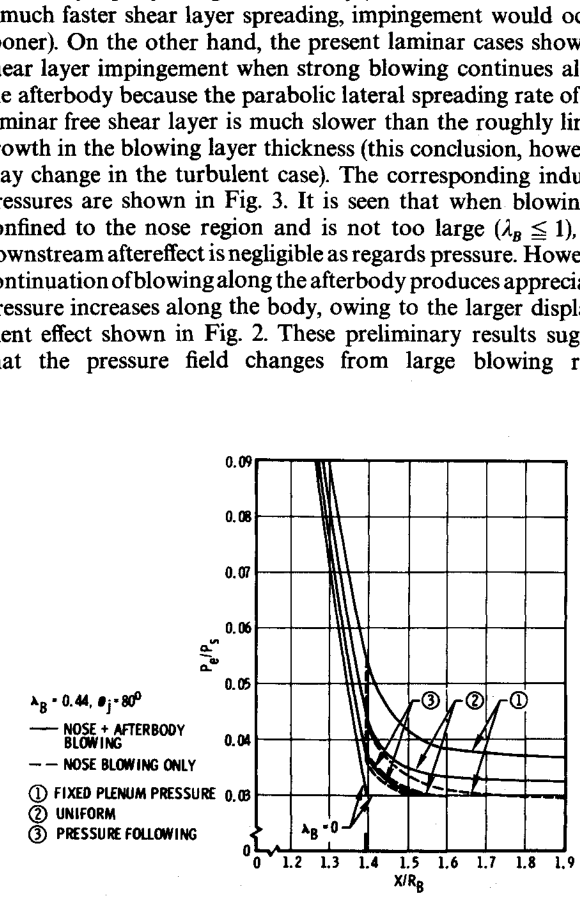

Fig. 3 Induced pressure distributions.

($\lambda_B \gg 1$) could significantly influence the aerodynamic properties of a massively-ablating planetary entry vehicle. Further parametric studies and extension of the present approach to turbulent flow are in progress, including consideration of upstream influence effects on finite length bodies and comparisons with available exact numerical solutions for massive blowing.

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Cross-Flow Effects on the Boundary Layer in a Plane of Symmetry

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Nomenclature

A = constant in skin friction law (see Eq. 12); $A = 0.0128$ for turbulent flow, $A = 0.22$ for laminar flow
C_f = local skin friction coefficient $= \tau_w/\rho_e u_e^2$
K_θ = correction factor on momentum thickness due to cross flow (Eq. 18)
K_{c_f} = correction factor due to cross flow (Eq. 17)
I, J, K = unit vectors in cartesian coordinates X, Y, Z (Fig. 1)
$\hat{i}, \hat{j}, \hat{k}$ = unit vectors in streamwise coordinates (Fig. 1)
e_v = unit vector in freestream direction (Fig. 1)
$\hat{m}$ = unit vector normal to both e_v and $\hat{n}$ (Fig. 1)
$\hat{n}$ = unit vector normal to surface (Fig. 1)
M = Mach number
n = exponent in skin friction law (Eq. 12), $n = 1.0$ for laminar flow, $n = 0.25$ for turbulent flow
p = pressure
p_2 $= \frac{1}{2}\partial^2 p/\partial\phi^2$ (Eq. 5)
$\dot{q}_w$ = heat-transfer rate at the wall
R = transverse radius of curvature of the body (Fig. 1)
R_{e_θ} = Reynolds number based on momentum thickness $= \rho_e u_e \Theta/\mu_e$
u_e = streamwise external velocity along x
v = cross-flow velocity derivative $= \partial w/\partial\phi$
V = total velocity
w = cross-flow velocity
x, y, z = Cartesian coordinates in streamline direction (Fig. 1)
X, Y, Z = Cartesian coordinates in body axes (Fig. 1)
α = angle of attack
δ = boundary layer thickness
δ^* = displacement thickness
θ = angle between the freestream velocity vector and normal to the surface (Fig. 1)
θ_c' = angle that the generatrix of the conical body makes with its axis
θ_0 = value of the angle θ in the plane of symmetry ($\phi = 0$)
Θ = momentum thickness
μ = viscosity
ϕ = meridional angle (Fig. 1)
ρ = density
τ_w = shear stress at the wall
ω = cross-flow parameter (Eq. 2)

Subscripts

e = edge condition
∞ = freestream condition
0 = value at plane of symmetry, $\phi = 0$
x = derivative with respect to x
X = derivative with respect to X
ϕ = derivative with respect to ϕ
$*$ = reference enthalpy condition

AN engineering method for estimating the boundary-layer properties on bodies in hypersonic flow is the "effective cone analysis." In this analysis, the flow properties external to the boundary layer are assumed to be the same as those existing on an equivalent cone of half angle equal to the cone half angle plus the angle of attack. This technique has been found to be in error,[1] especially in laminar boundary-layer regions.

In the present Note, simple engineering formulas, valid for noncircular cones at angle of attack, are derived to show that the effective cone analysis accounts only for the increase in the local edge Reynolds number due to the angle of attack. A correction factor (which depends on the cross flow produced by the transverse curvature of the body and on the angle of attack) to the effective cone analysis gives good agreement with exact theory. The present analysis is valid for hypersonic flight speeds and small cross-flow approximations.

The integral equations are analogous to those presented in Refs. 2–4. For conical bodies, the streamwise gradients vanish and the streamwise integral momentum equation reduces to (see Eq. 1 of Ref. 4).

$$\frac{d\Theta}{dx}+\Theta\frac{d\ln R}{dx}+\frac{v_e}{Ru_e}\int_0^\delta \frac{\rho}{\rho_e}\frac{v}{v_e}\left(1-\frac{u}{u_e}\right)dz=\frac{\tau_w}{\rho_e u_e^2}=\frac{C_f}{2} \qquad (1)$$

Note that in Ref. 4 the term v/v_e is assumed constant.

For conical bodies, $dv_e/dx = 0$ and v_e is obtained from (see Eq. 3 of Ref. 4)

$$\omega = v_e/R_x u_e = -\tfrac{1}{2}\{1-[1-8p_2/(\rho_e u_e^2 R_x^2)]^{1/2}\} \qquad (2)$$

For small cross flow

$$v_e/u_e \simeq -2p_2/\rho_e u_e^2 R_x \qquad (3)$$

Where

$$p_2 = \tfrac{1}{2}\partial^2 p/\partial\phi^2 \qquad (4)$$

is calculated from Newtonian theory,

$$p = p_\infty + \rho_\infty V_\infty^2 \sin^2\theta \simeq p_0 + p_2\phi^2 \quad \text{for} \quad \phi \ll 1 \qquad (5)$$

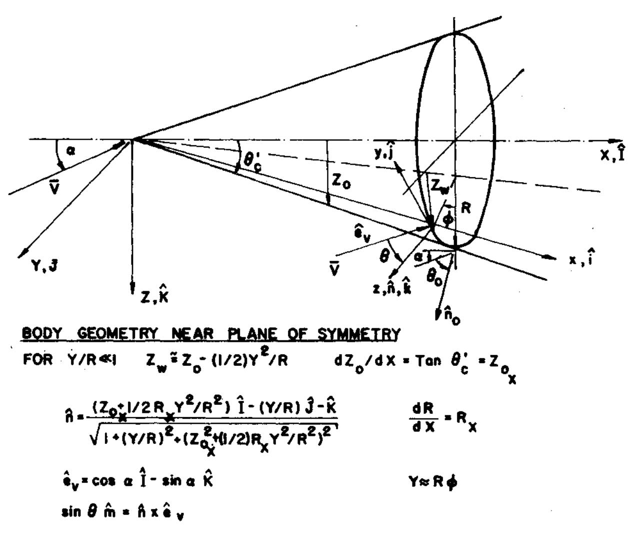

Fig. 1 Coordinate systems and body geometry.

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